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THREE TYPES OF FERMION MIXING AND POSSIBLE MANIFESTATIONS OF A PATI–SALAM LEPTOQUARK IN THE LOW–ENERGY PROCESSES

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Abstract

I report the recent studies^{1,2)} on the low-energy manifestations of a minimal extension of the Standard Model based on the quark-lepton symmetry $SU(4)_V \otimes SU(2)_L \otimes G_R$ of the Pati-Salam type. Given this symmetry the third type of mixing in the interactions of the $SU(4)_V$ leptoquarks with quarks and leptons is shown to be required. An additional arbitrariness of the mixing parameters could allow, in principle, to decrease noticeably the lower bound on the vector leptoquark mass originated from the low-energy rare processes. The decays $\mu \rightarrow e\gamma$, $\mu \rightarrow e\gamma\gamma$, and $\mu \rightarrow ee\bar{e}$ via the vector leptoquark are analysed and the specific hierarchy of the decay probabilities $\Gamma(\mu \rightarrow ee\bar{e}) \gg \Gamma(\mu \rightarrow e\gamma\gamma) \gg \Gamma(\mu \rightarrow e\gamma)$ is shown to exist. The upper limits on the branching ratios at a level of 10^{-18} for the $\mu \rightarrow e\gamma\gamma$ decay and at a level of 10^{-15} for the $\mu \rightarrow ee\bar{e}$ decay are established.

1 Introduction

All existing experimental data in particle physics are in good agreement with the Standard Model predictions. However, the problems exist which could not be resolved within the SM and it is obviously not a complete or final theory. It is unquestionable that the SM should be the low-energy limit of some higher symmetry. The question is what could be this symmetry. And the main question is what is the mass scale of this symmetry restoration. A gloomy prospect is the restoration of this higher symmetry at once on a very high mass scale, the so-called 'gauge desert'. A concept of a consecutive symmetry restoration is much more attractive. It looks natural in this case to suppose a correspondence of the hierarchy of symmetries and the hierarchy of the mass scales of their restoration. Now we are on the first step of some stairway of symmetries and we try to guess what could be the next one. If we consider some well-known higher symmetries from this point of view, two questions are pertinent. First, isn't the supersymmetry as the symmetry of bosons and fermions, higher than the symmetry within the fermion sector, namely, the quark-lepton symmetry³⁾, or the symmetry within the boson sector, namely, the left-right symmetry⁴⁾? Second, wouldn't the supersymmetry restoration be connected with a higher mass scale than the others?

We should like to analyse a possibility when the quark-lepton symmetry is the next step beyond the SM. We take a minimal symmetry of the Pati-Salam type with the lepton number as the fourth color³⁾, $SU(4)_V \otimes SU(2)_L \otimes G_R$. The fermions are combined into the fundamental representations of the $SU(4)_V$ group, the neutrinos with the *up* quarks and the charged leptons with the *down* quarks. Some attractive features of this symmetry should be pointed out.

i) The renormalizability of the SM demands some quark-lepton symmetry, namely, the fermions have to be combined into generations for the cancellation of the triangle anomalies.

ii) The proton decay is absent.

iii) A natural explanation for the quark fractional hypercharge takes place. Really, the 15-th generator of the $SU(4)$ group can be written in the form $T_{15} = \sqrt{3/8} \text{diag}(1/3, 1/3, 1/3, -1)$. It is traceless and the values of the left hypercharge Y_L appear to be placed on the diagonal. Let us call it the vector hypercharge, $Y_L = Y_V$.

iv) Let us suppose that $G_R = U(1)_R$. If we take the well-known values of the SM hypercharge of the left and right, and *up* and *down* quarks and leptons, then from the equation $Y_{SM} = Y_V + Y_R$ the values of the right hypercharge Y_R occur to be equal ± 1 for the *up* and *down* fermions, both quarks and leptons. It is tempting to interpret this fact as the evidence for the right hypercharge to be actually the doubled third component of the right isospin. Hence the G_R group is possibly $SU(2)_R$.

The most exotic object of the Pati-Salam type symmetry is the charged and colored gauge X boson named leptoquark. Its mass M_X should be the scale of reducing of $SU(4)_V$ to $SU(3)_c$. The bounds on the vector leptoquark mass⁵⁾ were obtained from the data on the $\pi \rightarrow e\nu$ decay and from the upper limit on $K_L^0 \rightarrow \mu e$ decay. In fact, these estimations were not comprehensive because the phenomenon of a mixing in the lepton-quark currents was not considered there. It can be shown that such a mixing inevitably occurs in the theory.

2 Three types of fermion mixing

Three fermion generations are combined into the $\{4,2\}$ representations of the semi-simple group $SU(4)_V \otimes SU(2)_L$ of the type

$$\begin{pmatrix} u^c & d^c \\ \nu & \ell \end{pmatrix}_i, \quad (i = 1, 2, 3), \quad (1)$$

where c is the color index. The mixing in the quark interaction with the W bosons being depicted by the Cabibbo-Kobayashi-Maskawa matrix is sure to exist in Nature. If one starts from the diagonal d, ν, ℓ states and the u states mixed by the CKM matrix than at the one-loop level the d states are mixed due to the conversion $d \rightarrow W + u(c, t) \rightarrow d'$ and then the ℓ states are mixed also, $\ell \rightarrow X + d(s, b) \rightarrow \ell'$, etc. Consequently, it is necessary for the renormalizability of the model to include all kinds of mixing at the tree-level. In the general case, none of the u, d, ν, ℓ components is the mass eigenstate. Due to the identity of the three representations (1) they always could be regrouped so that one of the components was diagonalized with respect to mass. If we diagonalize the charged lepton mass matrix, then the representations (1) can be rewritten to the form where the ν, u , and d states are not the mass eigenstates and are included into the same representations as the charged leptons $\ell, \nu_\ell = \mathcal{K}_{\ell i} \nu_i, u_\ell = \mathcal{U}_{\ell p} u_p, d_\ell = \mathcal{D}_{\ell n} d_n$. Here ν_i, u_p , and d_n ($i, p, n = 1, 2, 3$) are the mass eigenstates, and $\mathcal{K}_{\ell i}$, $\mathcal{U}_{\ell p}$, and $\mathcal{D}_{\ell n}$ are the unitary mixing matrices. The standard Cabibbo-Kobayashi-Maskawa matrix is seen to be $V = \mathcal{U}^+ \mathcal{D}$. This is as far as we know about \mathcal{U} and \mathcal{D} matrices. \mathcal{K} is the mixing matrix in the lepton sector.

3 Bounds from the low-energy experiments

Subsequent to the spontaneous $SU(4)_V$ symmetry breaking up to $SU(3)_c$ on the M_X scale six massive vector bosons are separated from the 15-plet of the gauge fields to generate three charged and colored leptoquarks. Their interaction with the fermions has the form

$$\mathcal{L}_X = \frac{g_S(M_X)}{\sqrt{2}} [\mathcal{D}_{\ell n} (\bar{\ell} \gamma_\alpha d_n^c) + (\mathcal{K}^+ \mathcal{U})_{ip} (\bar{\nu}_i \gamma_\alpha u_p^c)] X_\alpha^c + h.c. \quad (2)$$

The constant $g_S(M_X)$ can be expressed in terms of the strong coupling constant α_S at the leptoquark mass scale M_X , $g_S^2(M_X)/4\pi = \alpha_S(M_X)$.

If the momentum transferred is $q \ll M_X$, then the Lagrangian (2) in the second order leads to the effective four-fermion vector-vector interaction of quarks and leptons. By using the Fiertz transformation, the scalar,

pseudoscalar, vector and axial-vector terms may be separated in the effective Lagrangian. The QCD correction amounts to the appearance of the magnifying factor $Q(\mu)$ at the scalar and pseudoscalar terms, $Q(\mu) = (\alpha_S(\mu)/\alpha_S(M_X))^{4/\bar{b}}$. Here $\alpha_S(\mu)$ is the effective strong coupling constant at the hadron mass scale $\mu \sim 1 \text{ GeV}$, $\bar{b} = 11 - \frac{2}{3}\bar{n}_f$, \bar{n}_f is the averaged number of the quark flavors at the scales $\mu^2 \leq q^2 \leq M_X^2$. If the condition $M_X^2 \gg m_t^2$ is valid, then we have $\bar{n}_f \simeq 6$, and $\bar{b} \simeq 7$.

As the analysis shows, the tightest restrictions on the leptoquark mass M_X and the mixing matrix \mathcal{D} elements can be obtained from the experimental data on rare π and K decays and $\mu^- \rightarrow e^-$ conversion in nuclei. In the description of the interactions of π and K mesons it is sufficient to take the scalar and pseudoscalar terms only. As we shall see later, these terms acquire, in addition to the QCD corrections, an extra enhancement at the amplitude by the small quark current masses.

One can easily see that the leptoquark contribution to the $\pi \rightarrow e\nu$ decay is not suppressed by the electron mass in contrast to the W -contribution. Taking into account the interference of the leptoquark and W -exchange amplitudes we get the following expression for the ratio

$$R = \frac{\Gamma(\pi \rightarrow e\nu)}{\Gamma(\pi \rightarrow \mu\nu)} = R_{SM} \left[1 - \frac{2\sqrt{2}\pi \alpha_S(M_X) m_\pi^2 Q(\mu)}{G_F M_X^2 m_e (m_u(\mu) + m_d(\mu))} \operatorname{Re} \left(\frac{\mathcal{D}_{ed} \mathcal{U}_{eu}^*}{V_{ud}} \right) \right], \quad (3)$$

where $R_{SM} = (1.2345 \pm 0.0010) \cdot 10^{-4}$ is the value of the ratio in the Standard Model⁶⁾, $m_{u,d}(\mu)$ are the running current masses. To the $\mu \simeq 1 \text{ GeV}$ scale there correspond the well-known values $m_u \simeq 4 \text{ MeV}$, $m_d \simeq 7 \text{ MeV}$ and $m_s \simeq 150 \text{ MeV}$. Using the experimental data⁷⁾, $R^{exp} = (1.2310 \pm 0.0037) \cdot 10^{-4}$, we get the following lower bound on the leptoquark mass

$$M_X > (210 \text{ TeV}) \cdot |\operatorname{Re}(\mathcal{D}_{ed} \mathcal{U}_{eu}^* / V_{ud})|^{1/2}. \quad (4)$$

From the data on the $K \rightarrow e\nu$ decay we obtain similarly

$$M_X > (55 \text{ TeV}) \cdot |\operatorname{Re}(\mathcal{D}_{es} \mathcal{U}_{eu}^* / V_{us})|^{1/2}. \quad (5)$$

One can establish the following limits from the data⁸⁾ on the rare decays $K_L^0 \rightarrow \mu e$ and $K_L^0 \rightarrow e^+ e^-$

$$M_X > (1200 \text{ TeV}) \cdot |\mathcal{D}_{ed} \mathcal{D}_{\mu s}^* + \mathcal{D}_{es} \mathcal{D}_{\mu d}^*|^{1/2}, \quad (6)$$

$$M_X > (1400 \text{ TeV}) \cdot |Re(\mathcal{D}_{ed}\mathcal{D}_{es}^*)|^{1/2}. \quad (7)$$

The situation with another rare K decay, $K_L^0 \rightarrow \mu^+ \mu^-$, is rather intriguing. The recent measurements of the branching ratio at BNL⁹⁾ lowered its value closely to the unitary limit $Br_{abs} = 6.8 \cdot 10^{-9}$, and thus the decay amplitude has no real part. However, it was shown¹⁰⁾ that the real part could not be small in the SM with a heavy top quark. Isn't it a signal for a new physics, e.g. leptoquark? In this regard the discontinuance of the experiment KEK E137 where the $K_L^0 \rightarrow \mu^+ \mu^-$ decay rate was also measured, is regrettable.

A low-energy process under an intensive experimental searches, where the leptoquark could manifest itself is the μe conversion in nuclei. We estimate the branching ratio of the conversion in titanium and establish the bound on the model parameters on the base of the experimental data¹¹⁾

$$M_X > (680 \text{ TeV}) \cdot |\mathcal{D}_{ed}\mathcal{D}_{\mu d}^*|^{1/2}. \quad (8)$$

The above restrictions on the model parameters contain the elements of the unknown unitary mixing matrices \mathcal{D} and \mathcal{U} , which are connected by the condition $\mathcal{U}^+ \mathcal{D} = V$ only. Thus the possibility is not excluded, in principle, that the bounds obtained did not restrict M_X at all, e.g. if the elements \mathcal{D}_{ed} and $\mathcal{D}_{\mu d}$ were rather small. It would correspond to the connection of the τ lepton largely with the d quark in the \mathcal{D} matrix, and the electron and the muon with the s and b quarks. In general, it is not contradictory to anything even if it appears to be unusual. In this case a leptoquark could give a more noticeable contribution to the flavor-changing decays of the τ lepton and B mesons. However, an accuracy of these data is relatively poor. From the experimental limits¹²⁾ on the decays $\tau^- \rightarrow \mu^- K^0$, $\tau^- \rightarrow e^- K^0$, and $B^+ \rightarrow K^+ \mu^+ e^-$, $B^+ \rightarrow K^+ \mu^- e^+$, which are possible via the leptoquark exchange without suppression by the elements \mathcal{D}_{ed} and $\mathcal{D}_{\mu d}$, we obtain

$$M_X > (1 \text{ TeV}) \cdot |\mathcal{D}_{\mu s}\mathcal{D}_{\tau d}^*|^{1/2}, \quad M_X > (1 \text{ TeV}) \cdot |\mathcal{D}_{es}\mathcal{D}_{\tau d}^*|^{1/2}, \quad (9)$$

$$M_X > (2.4 \text{ TeV}) \cdot |\mathcal{D}_{es}\mathcal{D}_{\mu b}^*|^{1/2}, \quad M_X > (2.4 \text{ TeV}) \cdot |\mathcal{D}_{\mu s}\mathcal{D}_{eb}^*|^{1/2}. \quad (10)$$

In the recent paper¹³⁾ the limits on the Pati–Salam leptoquark were also considered in the specific cases when every charged lepton is connected with

only one quark in the currents. For the most part the results of ref.¹³⁾ agree with ours¹⁾.

4 Mixing-independent bound

We could find only one occasion when the mixing-independent lower bound on the leptoquark mass arises, namely, from the decay $\pi^0 \rightarrow \nu\bar{\nu}$. The best laboratory limit¹⁴⁾ on this decay is $Br(\pi^0 \rightarrow \nu\bar{\nu}) < 8.3 \cdot 10^{-7}$. In the papers¹⁵⁾ the almost coinciding cosmological and astrophysical estimations of the width of this decay were found: $Br(\pi^0 \rightarrow \nu\bar{\nu}) < 3 \cdot 10^{-13}$. Within the Standard Model this value is proportional to m_ν^2 . The process is also possible through the leptoquark mediation, without the suppression by the smallness of neutrino mass. On summation over all neutrino species the decay probability is mixing-independent. As a result the bound on the leptoquark mass occurs $M_X > 18 \text{ TeV}$. However, in the recent paper¹⁶⁾ a criticism has been expressed on both the cosmological and astrophysical limits. Therefore, only the laboratory limit¹⁴⁾ is reliable to establish the bound $M_X > 440 \text{ GeV}$.

5 Rare muon decays

The lepton-number violating decays $\mu \rightarrow e\gamma$, $\mu \rightarrow e\gamma\gamma$, $\mu \rightarrow ee\bar{e}$ are under the intensive experimental searches. Let us point out, however, that these decay modes are strongly suppressed in the SM with lepton mixing due to the well-known GIM cancellation¹⁷⁾ by the factor $(m_\nu/m_W)^4 \sim 10^{-39} \cdot (m_\nu/20 \text{ eV})^4$.

These processes arise in the model with vector leptoquark at the loop level via the virtual d, s, b quarks²⁾. As the analyses of the radiative muon decays show, the two-photon decay dominates the one-photon decay

$$\frac{\Gamma(\mu \rightarrow e\gamma\gamma)}{\Gamma(\mu \rightarrow e\gamma)} \sim \frac{\alpha}{\pi} \left(\frac{M_X}{m_b} \right)^4 \gg 1. \quad (11)$$

The magnitude of the $\mu \rightarrow e\gamma\gamma$ decay width could be estimated using the bound (8):

$$Br(\mu \rightarrow e\gamma\gamma) < 1.0 \cdot 10^{-18}. \quad (12)$$

A similar analysis of the $\mu \rightarrow ee\bar{e}$ decay within the above restrictions on the model parameters provides

$$Br(\mu \rightarrow ee\bar{e}) < 1.0 \cdot 10^{-15}. \quad (13)$$

Although the predicted values of the branches of the $\mu \rightarrow e\gamma\gamma$ and $\mu \rightarrow ee\bar{e}$ decays are essentially less than the existing experimental limits¹⁸⁾ $Br(\mu \rightarrow e\gamma\gamma)_{exp} < 7.2 \cdot 10^{-11}$, $Br(\mu \rightarrow ee\bar{e})_{exp} < 1.0 \cdot 10^{-12}$, they are not as small as the predictions of the SM with lepton mixing, and a hope for their observation in the future still remains.

6 Conclusions

- The bounds on the Pati–Salam leptoquark mass were reexamined with taking account of the mixing in the quark–lepton currents.
- Some semileptonic meson decays strongly suppressed within the SM could be induced by vector leptoquark. Their further experimental investigations are very important.
- The only mixing independent bound on the vector leptoquark mass arises from the limits on the invisible $\pi^0 \rightarrow \nu\bar{\nu}$ decay. It is $M_X > 440$ GeV from the laboratory limit and $M_X > 18$ TeV from the cosmological limit, but the last has to be verified.
- The specific hierarchy of the rare muon decay probabilities via vector leptoquark takes place and the branching ratios are not as small as the predictions of the SM with lepton mixing.

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